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# Book Review

T. S. CHIHARA, *An Introduction to Orthogonal Polynomials*, Gordon & Breach, New York/London/Paris, 1978, 249 pp.

At the Bar-le-Duc meeting on orthogonal polynomials W. Gautschi and W. Hahn both remarked on the review of Chihara's book which appeared in *Zentralblatt für Mathematik* **389** (1979), 114 (#33008). They thought it was a poor review. I had missed it, but after returning to Madison I looked it up. The reviewer read a different book than I did. Here is my opinion.

A couple of reviewers of this book think the reason for the renewal of interest in orthogonal polynomials is current work in approximation theory and numerical analysis. That is false. Splines have taken over the role that polynomials used to play in approximation theory, and while orthogonal polynomials and Gaussian quadrature play a small role in the use of splines, this would not account for the wide interest in orthogonal polynomials. Actually there are a number of reasons for the interest in orthogonal polynomials. General orthogonal polynomials are primarily interesting because of their 3-term recurrence relation. The first four chapters of Chihara's book treat orthogonal polynomials from the point of view of their recurrence relation, and of what conditions on the coefficients imply about the weight function. After treating the standard general theorems, the author uses some results of Stieltjes to study the case when the measure is supported on a half line. These methods, which the author uses in the form of chain sequences, are powerful enough so the author is able to construct a set of orthogonal polynomials whose recurrence relation is

$$2xp_n(x) = p_{n+1}(x) + C_n P_{n-1}(x)$$

with $C_n = 1 + O(n^{-2})$, and the measure has an absolutely continuous part on $[-1, 1]$ and infinitely many mass points outside $[-1, 1]$. Others had claimed there could only be finitely many mass points outside $[-1, 1]$ when $C_n = 1 + O(n^{-2})$. This claim was in print three years before this book appeared, and the author knew his example a couple of years before publication of this book. The reason this is not included is that the book was accepted for publication by the publisher in 1970 or 1971, but was not published until 1978. Some interesting work was done in this interval, but the deeper work of Nevai and his coworkers did not start to appear until 1979. Nevai's work is one of the real reasons there is a lot of work being done on general orthogonal polynomials. The problems were always interesting; what was missing were ideas about how to attack them. The first two-thirds of Chihara's book is still a nice introduction to general orthogonal polynomials, but it is not nearly as good a summary of known methods as it was when it was written about fifteen years ago.

The last two chapters summarize some of what Chihara knew about explicit sets of orthogonal polynomials. Chihara knows the literature on specific sets of orthogonal polynomials very well, and these two chapters are a gold mine for those of us who want to study specific sets of orthogonal polynomials. There are a few sets that were known in 1970 that are not mentioned. The most important was found by L. J. Rogers in 1895, and the only mention of these polynomials is in a cryptic reference to Allaway's thesis, where these polynomials were found for the fourth time. However in the early 1970s no one understood these polynomials, so it is not surprising they were not included. Many of the polynomials given in these two chapters can now be put into the chart of classical hypergeometric orthogonal polynomials (those on the Tableau d'Askey) or in the basic hypergeometric ver-



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sion of this table which has not been made yet. However, there are many nonclassical polynomials given in these two chapters; and some of them, like the Pollaczek polynomials, are very important.

In summary, a much better version of this book could be written now, but when it was written in the late 1960s and even when it was published in 1978 it was an excellent summary of what the author set out to do. And what he wanted to do was the right thing at that time. He did not treat much of the material covered in Freud's book on orthogonal polynomials; there was no need to. Together these two books are a good supplement to, but not a replacement for, Szegö's great book on orthogonal polynomials.

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